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# Quark distributions in a medium

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## Abstract

We derive the formal expressions needed to discuss the change of the twist-two parton distribution functions when a hadron is placed in a medium with relativistic scalar and vector mean fields.

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## 1. Introduction

Presently, the quark distributions of the free proton are quite well known over a wide range of  $x$  and  $Q^2$ . However, the theoretical understanding of these distributions is somewhat limited. Although we know how to extract the quark distributions from the measured structure functions within a NLO QCD analysis, we do not have sufficient control of non-perturbative QCD to calculate these distributions from first principles. Nevertheless, we do know how to formulate the problem in terms of the proton matrix elements of certain local operators (see, for instance, Ref. [1]), which correspond to the moments of the measured parton distributions. There has been considerable progress in calculating at least the first few moments using lattice QCD [2]—albeit at relatively large quark masses [3].

When it comes to nuclear structure functions one needs to evaluate the matrix elements of these same operators in the nuclear ground state—a priori a much more difficult problem. On the other hand, one knows that nuclear structure functions (divided by the number of constituent nucleons) lie within 10–20% of the free nucleon structure function (except near the kinematic boundary for the free nucleon) [4]. It therefore seems reasonable to tackle the problem by computing the corrections to the structure function of a bound nucleon and then allowing for Fermi motion. In particular, one could consider as a starting point the case of infinite quark matter. Even this presents serious theoretical challenges, because one knows from numerous studies that one encounters large scalar and vector mean-fields in this problem [5], and there has been no discussion of the formal aspects of parton distributions in such an environment (analogous to the discussion of Jaffe [1] in the free case).

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We therefore begin with a formal development of the parton model for the case of a “proton” embedded in constant scalar and vector fields. Since we are interested in modelling QCD, asymptotic freedom will be imposed by hand, in that between the two hard collisions which define the forward Compton amplitude (for the leading-twist parton distributions) the quark struck by the photon will be treated as free. The result of this formal investigation is of general interest as the formal properties of the parton distributions, in terms of support and reflectivity or crossing symmetry ( $x \rightarrow -x$ ), are guaranteed. Even so, it is possible to make suitable definitions of both the valence and sea-quark distributions. In [Section 2](#) we derive the equations for these quark distributions in a bound proton. In [Section 3](#) we verify that they are normalized. [Section 4](#) is devoted to the investigation of the effects that the nuclear medium exerts on the distributions, while in [Section 5](#) we apply our results to the case of infinite, isospin symmetric, quark matter. Finally, in [Section 6](#) we suggest directions for future work.

## 2. Quark and antiquark distributions

Our aim is to write down the formal expression for the quark and antiquark distributions of a bound proton. The in-medium proton momentum is denoted by  $P^* = (P^{*0}, \vec{P}^*)$ , where the star superindex means that nuclear interactions have produced an effective mass and energy for the proton. As in deep inelastic scattering there are two independent variables to build the hadronic tensor. We will use  $P^*$  and  $q$ , the photon momentum probing the nuclei, as those variables. Hence, the hadronic tensor for the bound proton is written as:

$$W_{\mu\nu}(P^*, q) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1^{\text{BP}}(x^*, q^2) + \left(P_\mu^* - \frac{P^* \cdot q}{q^2} q_\mu\right) \left(P_\nu^* - \frac{P^* \cdot q}{q^2} q_\nu\right) \frac{F_2^{\text{BP}}(x^*, q^2)}{P^* \cdot q}, \quad (1)$$

where  $F_1^{\text{BP}}(x^*, q^2)$  and  $F_2^{\text{BP}}(x^*, q^2)$  are the structure functions for the bound proton, and  $x^* = -q^2/2(P^* \cdot q)$  is the fraction of the bound proton momentum carried by the quarks. In the parton model, the structure functions are written as

$$2x^* F_1^{\text{BP}}(x^*, Q^2) = F_2^{\text{BP}}(x^*, Q^2) = \sum_{i=u,d,s,\dots} e_i^2 (q_i^{\text{BP}}(x^*) + \bar{q}_i^{\text{BP}}(x^*)) + O(\alpha_s(Q^2)), \quad (2)$$

where  $Q^2 = -q^2$ ,  $q_i^{\text{BP}}(x^*)$  ( $\bar{q}_i^{\text{BP}}(x^*)$ ) is the quark (antiquark) distribution in the bound proton and  $O(\alpha_s(Q^2))$  are the QCD corrections to the parton model.

In its simplest form, the quark distributions in the parton model are calculated from the handbag diagram. In the light-cone gauge,  $A^+ = 0$ , the distributions can be written as [\[1\]](#):

$$q^{\text{BP}}(x^*) = \frac{P^{*+}}{4\pi} \int dz^- e^{-ix^* P^{*+} z^-} \langle P^* | \psi_+^\dagger(z^-) \psi_+(0) | P^* \rangle_c \Big|_{z^+ = z^\perp = 0}, \quad (3)$$

where  $P^{*+} = (P^{*0} + P^{*3})/\sqrt{2}$  is the plus component of the bound proton momentum and  $|P^*\rangle$  is the state vector of the bound proton. If the quark field operators in [Eq. \(3\)](#) are expanded in terms of free plane waves,  $q^{\text{BP}}(x^*)$  can be rewritten as

$$q^{\text{BP}}(x^*) = \frac{P^{*+}}{2\pi} \int \frac{d^3 k}{(2\pi)^3} U_{\alpha\beta}(\vec{k}) \int dz^- e^{-ix^* P^{*+} z^- - i\vec{k} \cdot \vec{z}} \langle P^* | b_{\vec{k}}^{\alpha\dagger}(t) b_{\vec{k}}^\beta(0) | P^* \rangle_c \Big|_{z^+ = z^\perp = 0} \\ + \frac{P^{*+}}{2\pi} \int \frac{d^3 k}{(2\pi)^3} V_{\alpha\beta}(\vec{k}) \int dz^- e^{-ix^* P^{*+} z^- + i\vec{k} \cdot \vec{z}} \langle P^* | d_{\vec{k}}^\alpha(t) d_{\vec{k}}^{\beta\dagger}(0) | P^* \rangle_c \Big|_{z^+ = z^\perp = 0}, \quad (4)$$

where  $U_{\alpha\beta}(\vec{k}) = u_{(\alpha)}^\dagger(\vec{k})[1 + \gamma^0 \gamma^3] u_{(\beta)}(\vec{k})$ ,  $V_{\alpha\beta}(\vec{k}) = v_{(\alpha)}^\dagger(\vec{k})[1 + \gamma^0 \gamma^3] v_{(\beta)}(\vec{k})$ , with  $\alpha, \beta$  the polarization indices and the Einstein convention for repeated indices is understood. The time dependence of the creation and

annihilation operators is calculated from:

$$b_k^\alpha(t) = e^{i\hat{H}t} b_k^\alpha e^{-i\hat{H}t}, \quad (5)$$

with  $\hat{H}$  the QCD Hamiltonian operator. We assume that the state vector  $|P^*\rangle$  is an eigenstate of  $\hat{H}$  with eigenvalue  $P^{*0}$ . Hence, the insertion of a complete set of intermediate states in Eq. (4) implies that

$$q^{\text{BP}}(x^*) = \frac{P^{*+}}{2\pi} \int \frac{d^3k}{(2\pi)^3} U_{\alpha\beta}(\vec{k}) \int dz^- e^{-ix^*P^{*+}z^- + i(P^{*0} - P_n^{*0})t - i\vec{k}\cdot\vec{z}} \sum_n \langle P^* | b_k^{\alpha\dagger} | n \rangle \langle n | b_k^\beta | P^* \rangle_c \Big|_{z^+ = z^\perp = 0} \\ + \frac{P^{*+}}{2\pi} \int \frac{d^3k}{(2\pi)^3} V_{\alpha\beta}(\vec{k}) \int dz^- e^{-ix^*P^{*+}z^- - i(P_n^{*0} - P^{*0})t + i\vec{k}\cdot\vec{z}} \sum_n \langle P^* | d_k^\alpha | n \rangle \langle n | d_k^{\beta\dagger} | P^* \rangle_c \Big|_{z^+ = z^\perp = 0}, \quad (6)$$

where  $P_n^{*0}$  is the eigenvalue of the intermediate state  $|n\rangle$  after the action of the Hamiltonian operator. The integrals in  $z^-$  in Eq. (6) can be done:

$$\int dz^- e^{-ix^*P^{*+}z^- + i(P^{*0} - P_n^{*0})t - i\vec{k}\cdot\vec{z}} \Big|_{z^+ = z^\perp = 0} = \frac{2\pi}{P^{*+}} \delta\left(x^* - \frac{k_q^{*+}}{P^{*+}}\right), \quad (7)$$

$$\int dz^- e^{-ix^*P^{*+}z^- - i(P_n^{*0} - P^{*0})t + i\vec{k}\cdot\vec{z}} \Big|_{z^+ = z^\perp = 0} = \frac{2\pi}{P^{*+}} \delta\left(x^* + \frac{k_{\bar{q}}^{*+}}{P^{*+}}\right), \quad (8)$$

where  $k_q^{*+} = (P^{*0} - P_n^{*0} + k_q^3)/\sqrt{2}$  is the plus component for quarks and  $k_{\bar{q}}^{*+} = (P_n^{*0} - P^{*0} + k_{\bar{q}}^3)/\sqrt{2}$  for antiquarks. We then have:

$$q^{\text{BP}}(x^*) = \int \frac{d^3k}{(2\pi)^3} U_{\alpha\beta}(\vec{k}) \delta\left(x^* - \frac{k_q^{*+}}{P^{*+}}\right) \sum_n \langle P^* | b_k^{\alpha\dagger} | n \rangle \langle n | b_k^\beta | P^* \rangle_c \\ + \int \frac{d^3k}{(2\pi)^3} V_{\alpha\beta}(\vec{k}) \delta\left(x^* + \frac{k_{\bar{q}}^{*+}}{P^{*+}}\right) \sum_n \langle P^* | d_k^\alpha | n \rangle \langle n | d_k^{\beta\dagger} | P^* \rangle_c. \quad (9)$$

We note that the second term contributes to the quark distribution because an antiquark with a negative momentum fraction can be interpreted as a quark with positive momentum,  $k_{\bar{q}}^{*+} = -k_{\bar{q}}^{*-}$ .

If we assume that quarks in the nuclear environment feel scalar and vector mean fields then we can write the vector potential as a shift in the quark and antiquark energies:

$$k_q^{*+} = k_q^+ + V^+, \quad k_{\bar{q}}^{*+} = k_{\bar{q}}^+ - V^+, \quad (10)$$

where  $V^+$  is the plus component of the vector potential, and  $k_q^+$  ( $k_{\bar{q}}^+$ ) the plus component of quark (antiquark) momentum with masses modified by the scalar potential. Hence, from delta functions in Eq. (9) we have:

$$x^* P^{*+} = x P^+ + V^+, \quad (11)$$

where  $x = k_q^+/P^+$  is the fraction of momentum carried by a quark inside a proton immersed in a medium with scalar but with no vector mean field. With the help of Eq. (11) we can isolate the effect of the vector potential on the quark distributions:

$$q^{\text{BP}}(x^*) = (P^{*+}/P^+) q^{\text{BP}}(x), \quad (12)$$

with the bound quark distribution without the vector potential defined as

$$q^{\text{BP}}(x) = \frac{P^+}{4\pi} \int dz^- e^{-ixP^+z^-} \langle P^* | \psi_+^\dagger(z^-) \psi_+(0) | P^* \rangle_c \Big|_{z^+ = z^\perp = 0}. \quad (13)$$

As the effect of the vector mean field only changes the phase of the eigenstates, we conserved the notation  $|P^*\rangle$  for the bound proton with no vector potential when defining  $q^{\text{BP}}(x)$ . Finally, because the scalar potential can be absorbed in the proton mass, it follows that the whole analysis of Jaffe [1] for the support of the Bjorken  $x_{\text{Bj}}$  can be translated to the present case, meaning that the support for  $x$  is  $0 \leq x \leq 1$ . Similar relations to Eqs. (11) and (12) were also obtained by Mineo et al. [6], where the vector potential was treated as a gauge transformation.

On the other hand, the antiquark distribution in the parton model is given by the following expression [1]:

$$\bar{q}^{\text{BP}}(x^*) = -\frac{P^{*+}}{4\pi} \int dz^- e^{-ix^* P^{*+} z^-} \langle P^* | \psi_+^\dagger(0) \psi_+(z^-) | P^* \rangle_c \Big|_{z^+=z^-=0}. \quad (14)$$

Inserting a complete set of intermediate states and performing the integrals over the  $z^-$  variable, as in Eqs. (7) and (8), we find the following expression for the antiquark distributions:

$$\begin{aligned} \bar{q}^{\text{BP}}(x^*) = & \int \frac{d^3k}{(2\pi)^3} U_{\alpha\beta}(\vec{k}) \delta\left(x^* + \frac{k_q^{*+}}{P^{*+}}\right) \sum_n \langle P^* | b_k^\alpha | n \rangle \langle n | b_k^{\beta\dagger} | P^* \rangle_c \\ & + \int \frac{d^3k}{(2\pi)^3} V_{\alpha\beta}(\vec{k}) \delta\left(x^* - \frac{k_{\bar{q}}^{*+}}{P^{*+}}\right) \sum_n \langle P^* | d_k^{\beta\dagger} | n \rangle \langle n | d_k^\alpha | P^* \rangle_c. \end{aligned} \quad (15)$$

The expressions for the quark and antiquark distributions, as given in Eqs. (9) and (15), have (as they should) the crossing symmetry:  $q^{\text{BP}}(x^*) = -\bar{q}^{\text{BP}}(-x^*)$ .

### 3. Normalization

The quark distributions of the bound proton must have the correct normalization. That is, the integral of  $q^{\text{BP}}(x^*) - \bar{q}^{\text{BP}}(x^*)$  over the allowed  $x^*$  range must give the number of valence quarks of the bound proton. To this end, we will use Eq. (11), together with the allowed support for  $x$ . It follows that the maximum value for  $x^*$  is  $1 - 2V^+$ , while the minimum value of  $x^*$  is  $V^+$ :

$$\begin{aligned} \int_{V^+}^{1-2V^+} [q^{\text{BP}}(x^*) - \bar{q}^{\text{BP}}(x^*)] dx^* &= \frac{P^{*+}}{2\pi} \int_{-\infty}^{\infty} dx^* dz^- e^{-ix^* P^{*+} z^-} \langle P^* | \psi_+^\dagger(z^-) \psi_+(0) | P^* \rangle_c \\ &= \langle P^* | \psi_+^\dagger(0) \psi_+(0) | P^* \rangle_c, \end{aligned} \quad (16)$$

which gives the quark number in the state  $|P^*\rangle$ :

$$\int_{V^+}^{1-2V^+} [q^{\text{BP}}(x^*) - \bar{q}^{\text{BP}}(x^*)] dx^* = N_q^{\text{BP}} - N_{\bar{q}}^{\text{BP}}. \quad (17)$$

### 4. The in-medium effects on the distributions

The quark and antiquark operators appearing in the distributions (9) and (15) are those of free quantum fields. The quarks which build the proton state are, however, not in free space: they are confined in a proton state, while the proton state is itself immersed in a nuclear medium. Our state vector has to be built from these bound quark operators. We shall denote by  $b_{\vec{p}}^*$  ( $d_{\vec{p}}^*$ ) the annihilation operator of a quark (antiquark) in the bound proton. Let the bound proton state be written as

$$|P^*\rangle = F[q_{\vec{p}_1}^{*\dagger}, q_{\vec{p}_2}^{*\dagger}, q_{\vec{p}_3}^{*\dagger}] |0^*\rangle, \quad (18)$$

where  $F[q_{\vec{p}_1}^{*\dagger}, q_{\vec{p}_2}^{*\dagger}, q_{\vec{p}_3}^{*\dagger}]$  is a functional of the bound quark operators,  $|0^*\rangle$  is the effective vacuum (as seen by quarks in a nuclear medium) where the quarks bound in the proton live, and  $\vec{p}_1, \vec{p}_2, \vec{p}_3$  are the individual momenta of the three valence quarks. Although only the valence quarks appear explicitly in the functional, it is understood that it may be populated by quark–antiquark pairs and gluons: the notation only shows the net number of quarks and antiquarks.

The effective vacuum is defined by:

$$b_{\vec{p}}^*|0^*\rangle = 0, \quad d_{\vec{p}}^*|0^*\rangle = 0. \quad (19)$$

With this definition, we can calculate the action of the free quark and antiquark operators on the bound proton state. To do this, note that the quark field operator in Eq. (3) could have been expanded in any basis [7]. Suppose that we know the solution of the Dirac equation for the interacting theory describing the bound proton state, with solutions  $u_{\text{BP}}(\vec{p}, \vec{x})$  ( $v_{\text{BP}}(\vec{p}, \vec{x})$ ) for the positive (negative) energy part. The field operator expanded in this basis is written as:

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3} \sum_{\alpha} [b_{\vec{p},\alpha}^*(t) u_{\text{BP}}^{(\alpha)}(\vec{p}, \vec{x}) + d_{\vec{p},\alpha}^{*\dagger}(t) v_{\text{BP}}^{(\alpha)}(\vec{p}, \vec{x})], \quad (20)$$

with  $u^\dagger u = 1$  [5]. If we compare it with the expansion in terms of the free fields, we get:

$$b_{\vec{k},\alpha}(t) = \int \frac{d^3p}{(2\pi)^3} \sum_{\beta} [b_{\vec{p},\beta}^*(t) A^{\alpha\beta}(\vec{k}, \vec{p}) + d_{\vec{p},\beta}^{*\dagger}(t) B^{\alpha\beta}(\vec{k}, \vec{p})], \quad (21)$$

where

$$A^{\alpha\beta}(\vec{k}, \vec{p}) = \int d^3x \bar{u}^{(\alpha)}(\vec{k}) \gamma_0 u_{\text{BP}}^{(\beta)}(\vec{p}, \vec{x}) e^{-i\vec{k}\cdot\vec{x}}, \quad (22)$$

$$B^{\alpha\beta}(\vec{k}, \vec{p}) = \int d^3x \bar{u}^{(\alpha)}(\vec{k}) \gamma_0 v_{\text{BP}}^{(\beta)}(\vec{p}, \vec{x}) e^{-i\vec{k}\cdot\vec{x}}. \quad (23)$$

The calculation of the anticommutator between the free and bound operators gives:

$$\{b_{\vec{k}}^{\alpha}(t), b_{\vec{p}}^{*\beta\dagger}(t')\}_{t=t'} = A^{\alpha\beta}(\vec{k}, \vec{p}), \quad (24)$$

and similar for the other operators. Thus the action of the free quark annihilation operator in the proton state results in

$$b_{\vec{k}}^{\beta}|P^*\rangle = A^{\beta\gamma}(\vec{k}, \vec{p}_i) G[b_{\vec{p}_j}^{*\dagger}, b_{\vec{p}_l}^{*\dagger}]^{\gamma}|0^*\rangle - F[b_{\vec{p}_1}^{*\dagger}, b_{\vec{p}_2}^{*\dagger}, b_{\vec{p}_3}^{*\dagger}] b_{\vec{k}}^{\alpha}|0^*\rangle, \quad (25)$$

where  $G[b_{\vec{p}_j}^{*\dagger}, b_{\vec{p}_l}^{*\dagger}]^{\gamma}$  is some function of the effective quark operators after  $b_{\vec{k}}^{\alpha}$  acts on  $F$ , and the index  $\gamma$  indicates that the resulting two quark states are in coloured states. Integration over  $\vec{p}_i, \vec{p}_j$  and  $\vec{p}_l$  is understood. Thus, the action of the free operator on the effective vacuum yields:

$$b_{\vec{k}}^{\beta}|0^*\rangle = \int \frac{d^3p}{(2\pi)^3} \sum_{\gamma} B^{\beta\gamma}(\vec{k}, \vec{p}) |\bar{q}_{\vec{p},\gamma}^* \rangle. \quad (26)$$

From Eqs. (25) and (26) we see that there will be contributions from two terms in the quark distribution, Eq. (9); one when we have two quarks in the intermediate state and the other when we have three quarks and one antiquark in the intermediate state. For the case  $n = 2$ , we define the coloured state:

$$|n = 2\rangle = G[b_{\vec{p}_j}^{*\dagger}, b_{\vec{p}_k}^{*\dagger}]^{\gamma}|0^*\rangle, \quad (27)$$

and for the case  $n = 4$ :

$$|n = 4\rangle = F[b_{\vec{p}_1}^{\dagger*}, b_{\vec{p}_2}^{\dagger*}, b_{\vec{p}_3}^{\dagger*}]|\vec{q}_{\vec{p}, \gamma}^*\rangle. \quad (28)$$

In a completely analogous way to what was done in Eqs. (21)–(23) for the free quark operator, we can express the free antiquark operator in terms of the bound quark and antiquark operators. We will call  $C^{\alpha\beta}(\vec{k}, \vec{p})$  the overlap analogous to the  $B^{\alpha\beta}(\vec{k}, \vec{p})$  term, with the interchange between the  $u$  and  $v$  Dirac spinors. Similarly,  $D^{\alpha\beta}(\vec{k}, \vec{p})$  will be the analogue of the  $A^{\alpha\beta}(\vec{k}, \vec{p})$  overlap, with the  $v$  Dirac spinor replacing the  $u$  Dirac spinor everywhere.

Using the states defined in Eqs. (27) and (28), the quark distribution Eq. (9) is finally written as

$$\begin{aligned} q^{\text{BP}}(x^*) = & \int \frac{d^3k}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} U_{\alpha\beta}(\vec{k}) \delta\left(x^* - \frac{k_q^{*+}}{P^{*+}}\right) \\ & \times \sum_{\gamma} (\langle n = 2 | n = 2 \rangle A^{\gamma\alpha\dagger}(\vec{k}, \vec{p}) A^{\gamma\beta}(\vec{k}, \vec{p}) - \langle n = 4 | n = 4 \rangle B^{\gamma\alpha\dagger}(\vec{k}, \vec{p}') B^{\gamma\beta}(\vec{k}, \vec{p})) \\ & + \int \frac{d^3k}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} V_{\alpha\beta}(\vec{k}) \delta\left(x^* + \frac{k_{\bar{q}}^{*+}}{P^{*+}}\right) \\ & \times \sum_{\gamma} (\langle n = 2 | n = 2 \rangle D^{\gamma\alpha\dagger}(\vec{k}, \vec{p}) D^{\gamma\beta}(\vec{k}, \vec{p}) - \langle n = 4 | n = 4 \rangle C^{\gamma\alpha\dagger}(\vec{k}, \vec{p}') C^{\gamma\beta}(\vec{k}, \vec{p})), \end{aligned} \quad (29)$$

where integration over all the internal momenta of the quarks in the intermediate states is understood.

## 5. The quark matter case

A central question from the point of view of nuclear physics involves the changes to the quark and antiquark distributions of a bound proton. Since one must develop a reliable model of both the free proton and the binding of nucleons starting from the quark level [8], this problem is rather complicated. We intend to report on our investigation of that problem in future work. For the present, we have chosen to illustrate the formal ideas developed here by applying them to a toy model, namely the quark distributions of isospin symmetric quark matter in which each quark feels a scalar potential,  $-V_s^q$ , and a vector potential,  $V_v^q$ . This is the premise of the Quark–Meson Coupling (QMC) model [9] which has been used successfully to calculate the properties of nuclear matter as well as finite nuclei [10,11]. Most recently it has also been used to derive an effective nuclear force which is very close to the widely used Skyrme III force [12]. (Except that in QMC the quarks are confined by the MIT bag, as well as feeling the mean-field scalar and vector potentials generated by the surrounding nucleons.) In the mean field approximation, the Dirac equation for the quark in infinite quark matter is written as:

$$[i\gamma \cdot \partial - (m - V_s^q) - \gamma_0 V_v^q] \psi_{\text{QM}}^q(x) = 0. \quad (30)$$

Following Eq. (20), we write the field operator in terms of the solutions of Eq. (30):

$$\psi_{\text{QM}}^q(x) = \int \frac{d^3p}{(2\pi)^3} \sum_{\alpha} [b_{\vec{p}, \alpha}^* u_{\text{QM}}^{(\alpha)}(\vec{p}) e^{-ip_q^{*0}t + i\vec{p} \cdot \vec{x}} + d_{\vec{p}, \alpha}^{*\dagger} v_{\text{QM}}^{(\alpha)}(\vec{p}) e^{ip_q^{*0}t - i\vec{p} \cdot \vec{x}}], \quad (31)$$

where  $u_{\text{QM}}^{(\alpha)}(\vec{p})$  and  $v_{\text{QM}}^{(\alpha)}(\vec{p})$  are the in-medium Dirac spinors with mass  $m^* = m - V_s^q$ , energy  $E_q^*$ , and where  $p_q^{*0} = E_q^* + V_v^q$  for quarks and  $p_{\bar{q}}^{*0} = E_q^* - V_v^q$  for antiquarks.

We now calculate Eq. (22) for the  $A$  factor, using  $k = (p_k^0, \vec{k})$  for the free quark and  $p = (p_q^{*0}, \vec{p})$  for the quark in infinite quark matter:

$$A^{\alpha\beta}(\vec{k}, \vec{p}) = \int d^3x \bar{u}^{(\alpha)}(\vec{k}) \gamma_0 u_{\text{QM}}^{(\beta)}(\vec{p}) e^{i(\vec{p} - \vec{k}) \cdot \vec{x}}. \quad (32)$$

The integral over  $\vec{x}$  can be done, giving a Dirac delta function, which implies that:

$$A^{\alpha\beta}(\vec{k}, \vec{p}) = (2\pi)^3 \delta(\vec{p} - \vec{k}) \bar{u}^{(\alpha)}(\vec{k}) \gamma_0 u_{\text{QM}}^{(\beta)}(\vec{p}). \quad (33)$$

Similarly, Eq. (23) for the  $B$  factor becomes:

$$B^{\alpha\beta}(\vec{k}, \vec{p}) = (2\pi)^3 \delta(\vec{p} + \vec{k}) \bar{u}^{(\alpha)}(\vec{k}) \gamma_0 v_{\text{QM}}^{(\beta)}(\vec{p}). \quad (34)$$

If we go back to Eq. (21) relating the free and bound quark operators, we find:

$$b_{\vec{k}}^{\alpha}(t) = \bar{u}^{(\alpha)}(\vec{k}) \gamma_0 u_{\text{QM}}^{(\beta)}(\vec{k}) b_{\vec{k},\beta}^{*}(t) + \bar{u}^{(\alpha)}(\vec{k}) \gamma_0 v_{\text{QM}}^{(\beta)}(-\vec{k}) d_{-\vec{k},\beta}^{*\dagger}(t). \quad (35)$$

A similar calculation for the antiquark operator gives:

$$d_{\vec{k}}^{\alpha\dagger}(t) = \bar{v}^{(\alpha)}(\vec{k}) \gamma_0 u_{\text{QM}}^{(\beta)}(-\vec{k}) b_{-\vec{k}}^{*\beta}(t) + \bar{v}^{(\alpha)}(\vec{k}) \gamma_0 v_{\text{QM}}^{(\beta)}(\vec{k}) d_{\vec{k}}^{*\dagger\beta}(t). \quad (36)$$

An explicit calculation shows that  $D^{\alpha\beta}(\vec{k}, \vec{p}) = A^{\alpha\beta}(\vec{k}, \vec{p})$  and  $C^{\alpha\beta}(\vec{k}, \vec{p}) = B^{\alpha\beta}(\vec{k}, \vec{p})$ , which implies that charge conjugation holds between the bound quark and antiquark operators.

Although the quark and antiquark operators in Eqs. (35) and (36) are time dependent, the expressions for the quark distributions involve products like  $b^{\dagger}b$ , which are time independent:

$$b_{\vec{k}}^{\dagger} b_{\vec{k}} = b_{\vec{k}}^{\dagger}(t) b_{\vec{k}}(t). \quad (37)$$

The bound quark distribution, calculated from Eq. (9), is then:

$$\begin{aligned} q^{\text{BP}}(x^*) &= \int \frac{d^3k}{(2\pi)^3} U^{\alpha\beta}(\vec{k}) \delta\left(x^* - \frac{k_q^{*+}}{P^{*+}}\right) \frac{E_q + m}{2E_q} \frac{E_q^* + m^*}{2E_q^*} \\ &\quad \times \left( \left[ 1 + \frac{\vec{k}^2}{(E_q + m)(E_q^* + m^*)} \right]^2 \langle P^* | b_{\vec{k},\alpha}^{*\dagger} b_{\vec{k},\beta}^* | P^* \rangle \right. \\ &\quad \left. + \left[ \frac{1}{E_q + m} - \frac{1}{E_q^* + m^*} \right]^2 (\chi_{\gamma}^{\dagger} \vec{\sigma} \cdot \vec{k} \chi_{\alpha}) (\chi_{\beta}^{\dagger} \vec{\sigma} \cdot \vec{k} \chi_{\delta}) \langle P^* | d_{-\vec{k}}^{*\gamma} d_{-\vec{k}}^{*\dagger\delta} | P^* \rangle \right) \\ &\quad + \int \frac{d^3k}{(2\pi)^3} V^{\alpha\beta}(\vec{k}) \delta\left(x^* + \frac{k_{\bar{q}}^{*+}}{P^{*+}}\right) \frac{E_q + m}{2E_q} \frac{E_q^* + m^*}{2E_q^*} \\ &\quad \times \left( \left[ 1 + \frac{\vec{k}^2}{(E_q + m)(E_q^* + m^*)} \right]^2 \langle P^* | d_{\vec{k},\alpha}^* d_{\vec{k},\beta}^{*\dagger} | P^* \rangle \right. \\ &\quad \left. + \left[ \frac{1}{E_q + m} - \frac{1}{E_q^* + m^*} \right]^2 (\chi_{\gamma}^{\dagger} \vec{\sigma} \cdot \vec{k} \chi_{\alpha}) (\chi_{\beta}^{\dagger} \vec{\sigma} \cdot \vec{k} \chi_{\delta}) \langle P^* | b_{-\vec{k}}^{*\gamma\dagger} b_{-\vec{k}}^{*\delta} | P^* \rangle \right). \end{aligned} \quad (38)$$

## 6. Concluding remarks

We have developed the formal framework for evaluating the change in the structure function of a hadron when it is bound in a nuclear medium. In particular, the formalism is able to deal with the separate changes in the quark and antiquark distributions, while preserving the necessary sum rules. Since much of our information on the parton distribution functions of the nucleon does in fact come from nuclear data this is an especially important issue [13–15]. In order to illustrate the formalism we considered the parton distribution functions for quark matter with mean field scalar and vector potentials. In the immediate future it will be important to include the effect of confinement, as for example in the QMC model [9,16] or even the NJL model [6,17] with proper time regularization [18,19].

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